QCD phase transition basics

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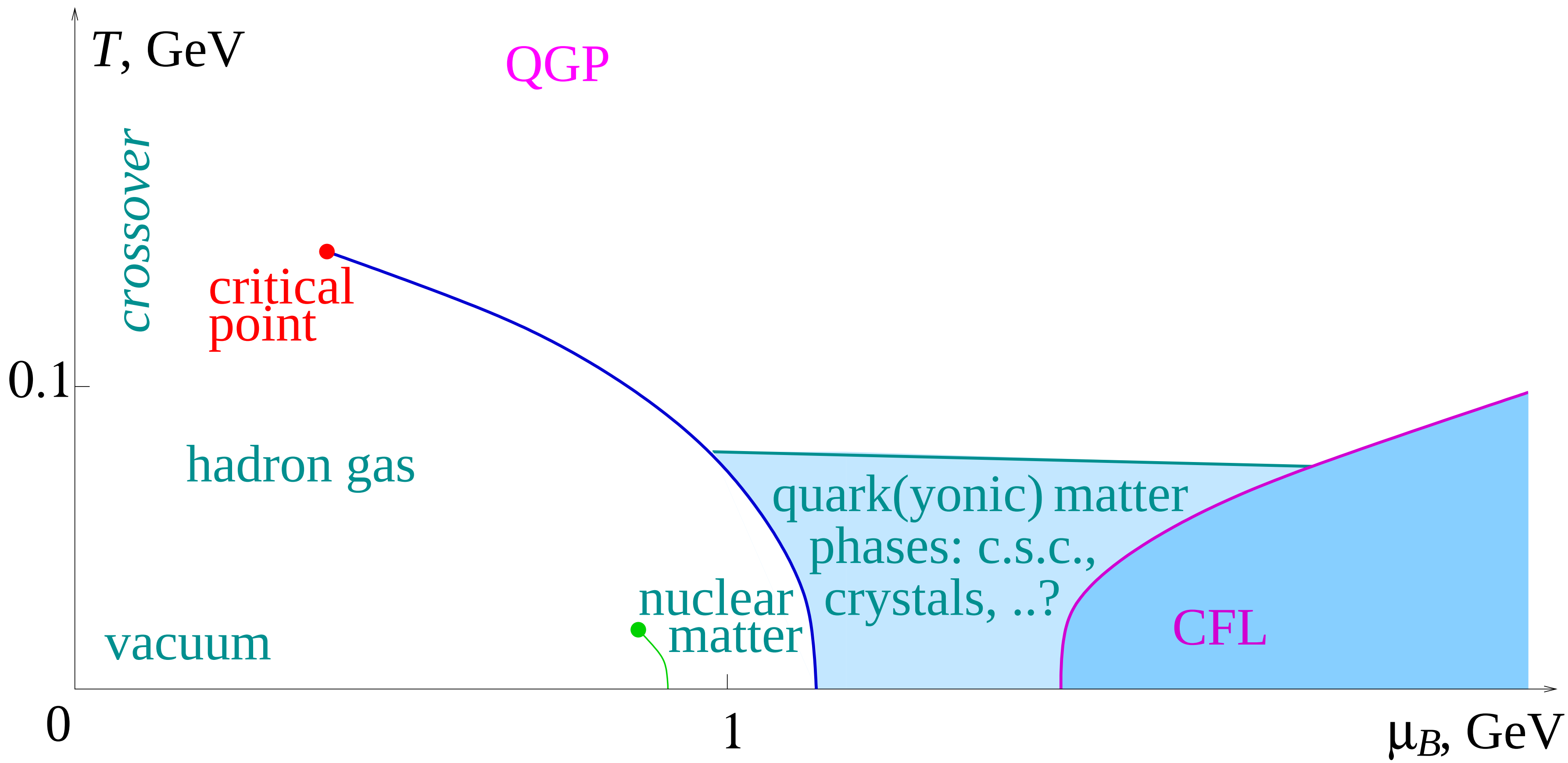
QCD phase diagram (today)



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- Can we observe the critical point in heavy ion collisions, and how?

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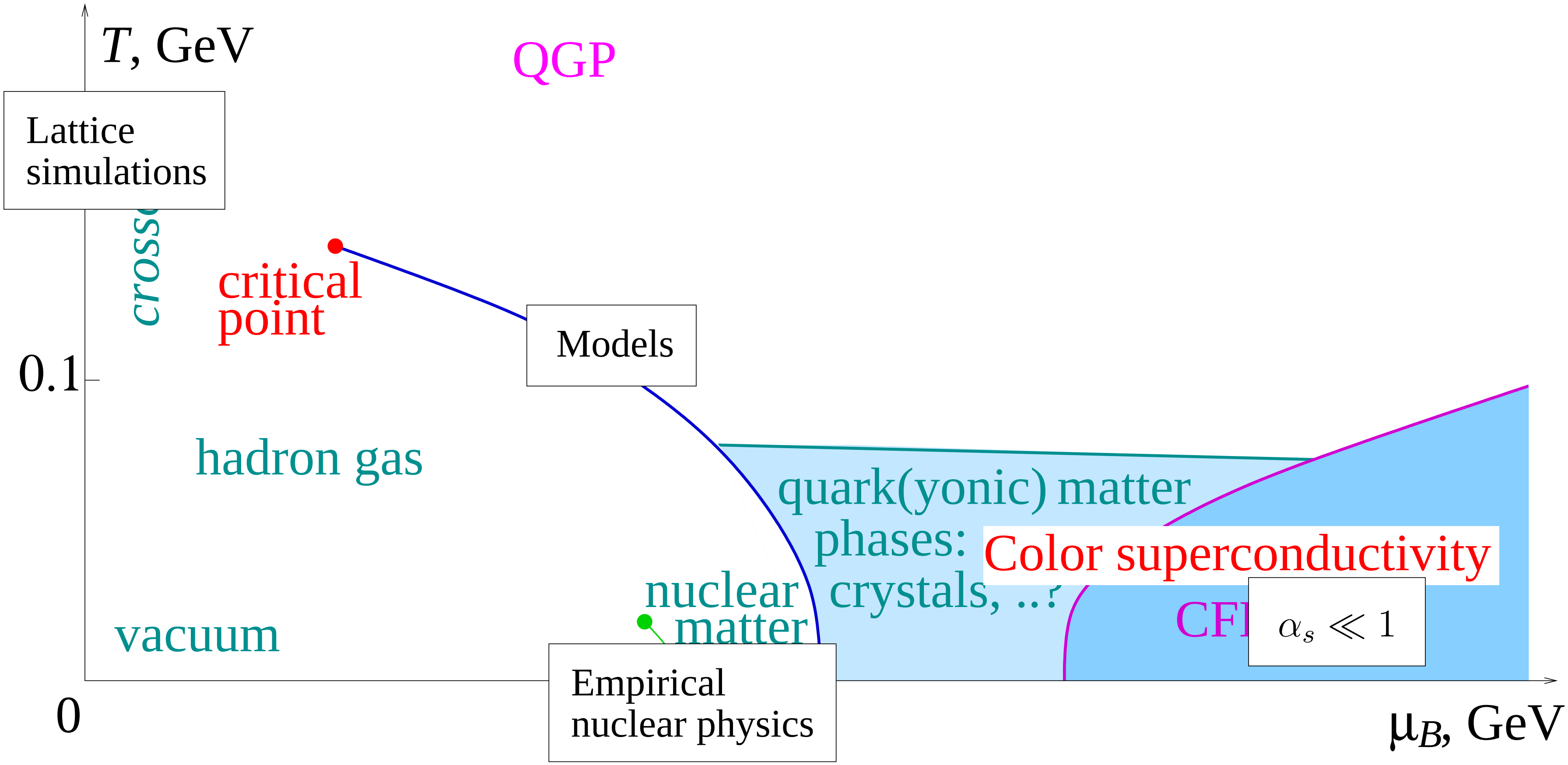

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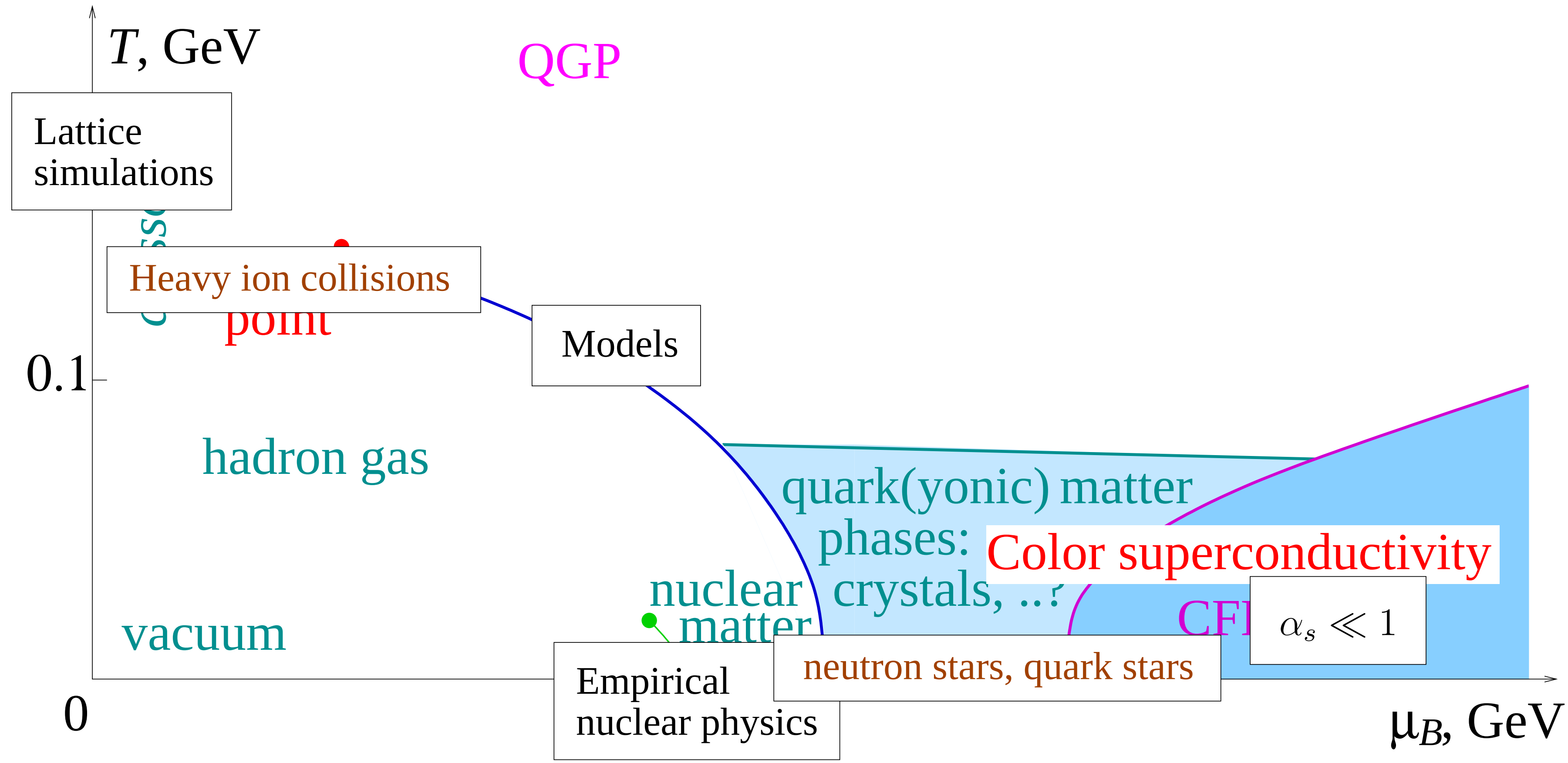

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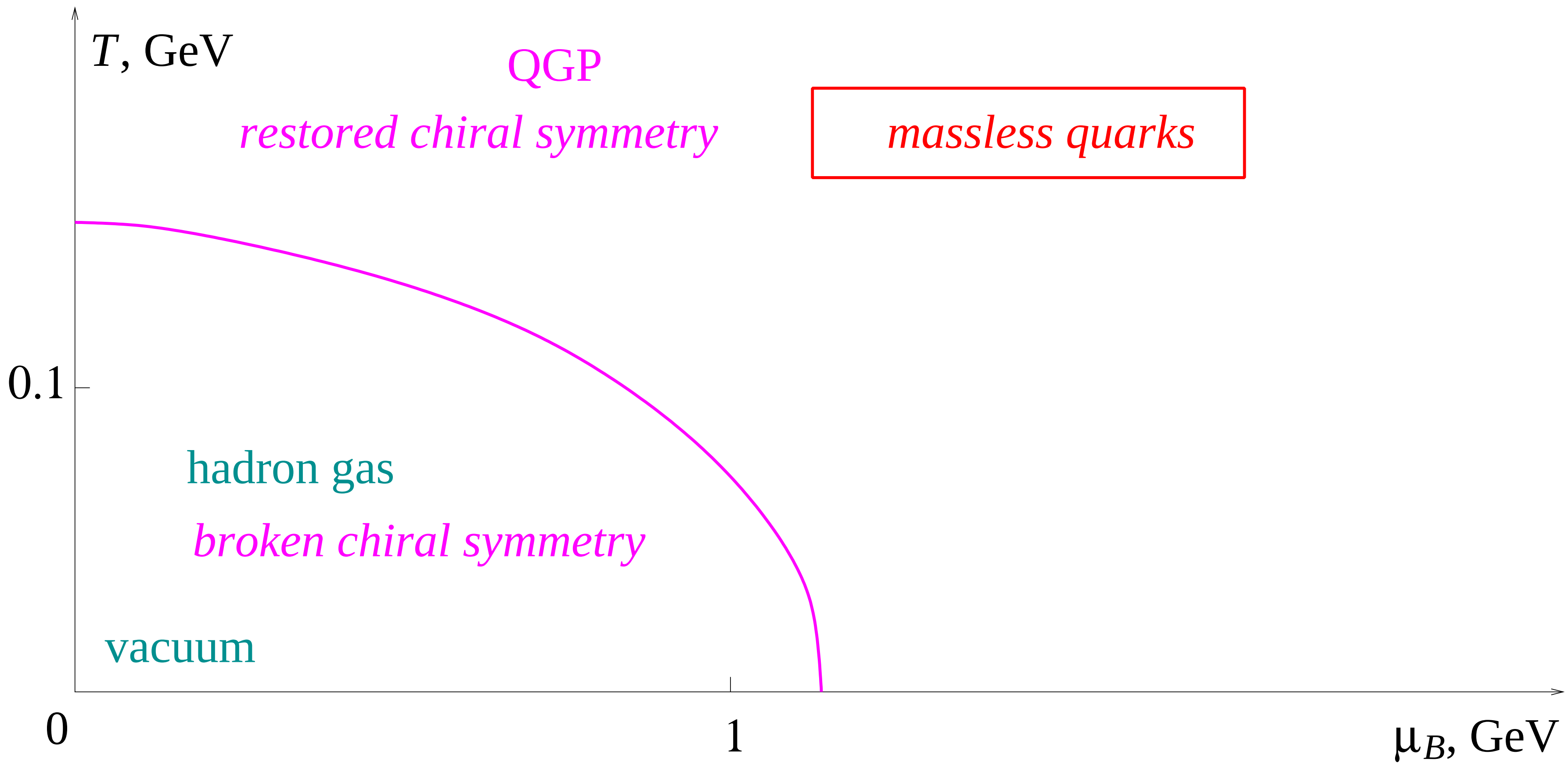
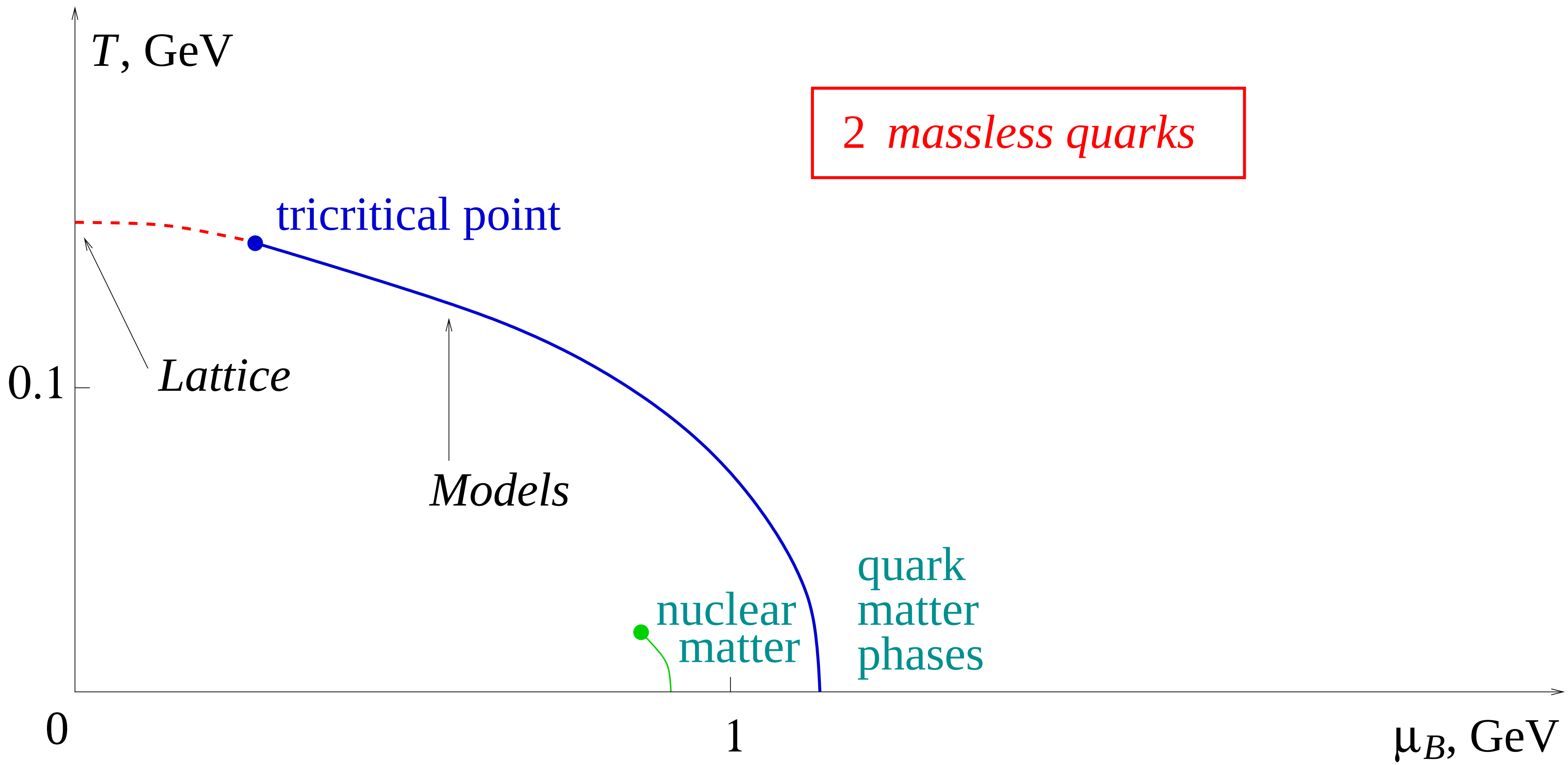
QCD phase diagram (role of the chiral symmetry)



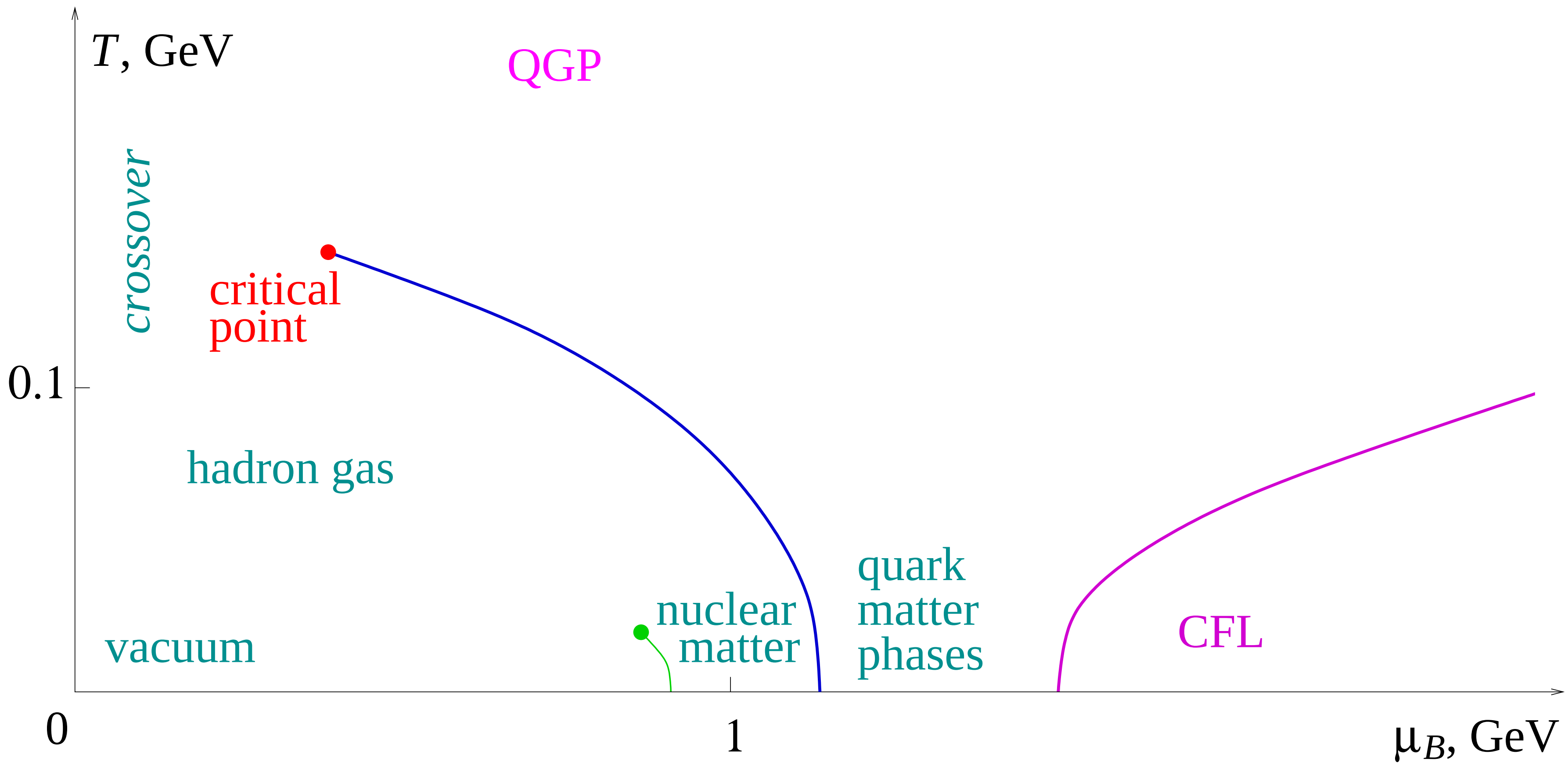
What is the order of the transition?

QCD phase diagram (role of the chiral symmetry)



Note: nuclear matter is on the chirally broken side.

QCD phase diagram (role of the chiral symmetry)



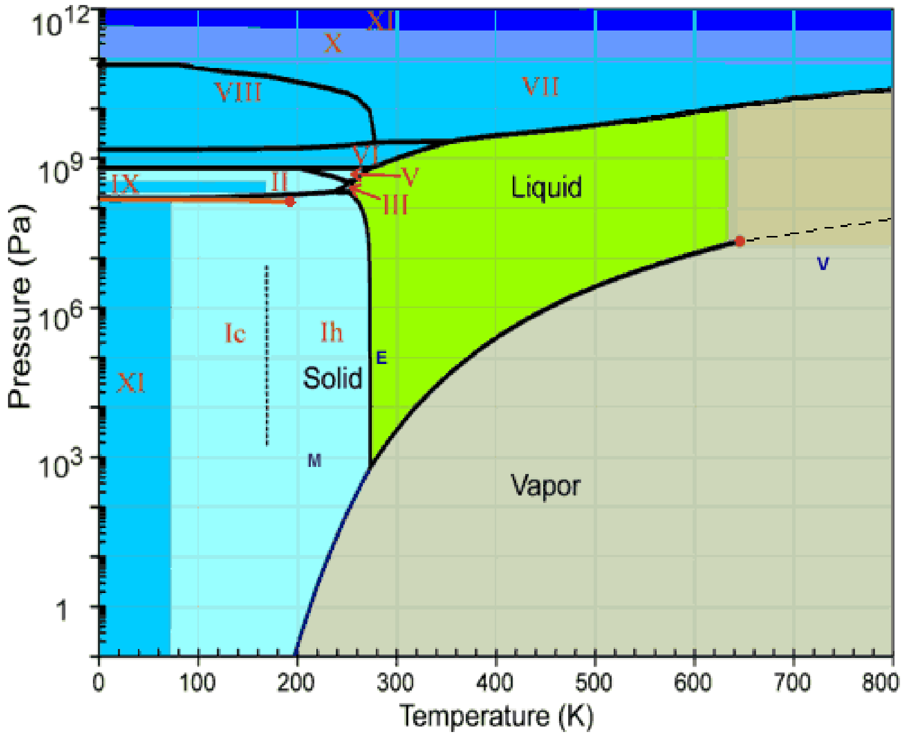
Lattice: Crossover is firmly established (most recently Aoki *et al*)

RHIC: Matter near/above crossover – strongly coupled liquid. LHC will study it.

Critical points in known liquids

Critical point ∃ in many liquids (critical opalescence).

Water:



Confinement/deconfinement transition

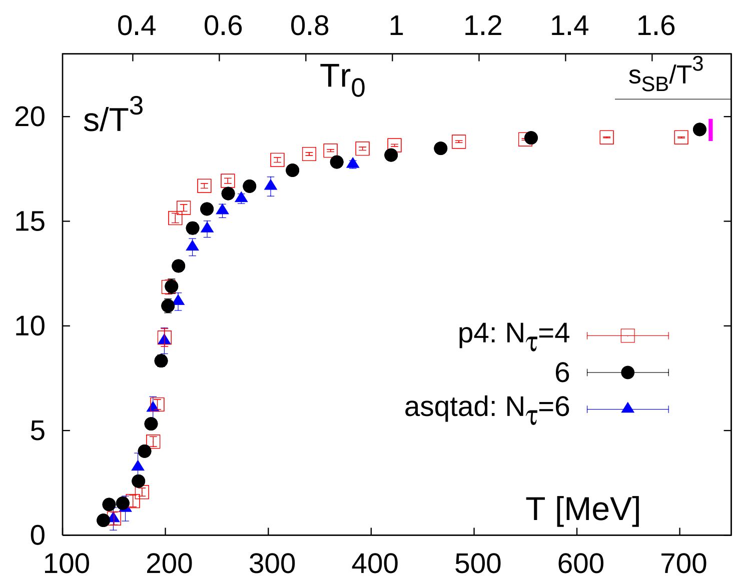
- Confinement is difficult to define for theories with quarks.
  - Polyakov's definition, $\langle P \rangle = 0$, does not work, because $\langle P \rangle \neq 0$. The Z_3 symmetry is out once quarks are in.
  - Confining string between two color sources is *not* infinite — it snaps:

$$Q - - - - - - \bar{Q} \qquad \Rightarrow \qquad Q - - \bar{q} \quad + \quad q - - \bar{Q}$$

  - "No colored states"? This is true by *definition* of the theory. Not a dynamical property. There is no *de*confinement in this definition of confinement.

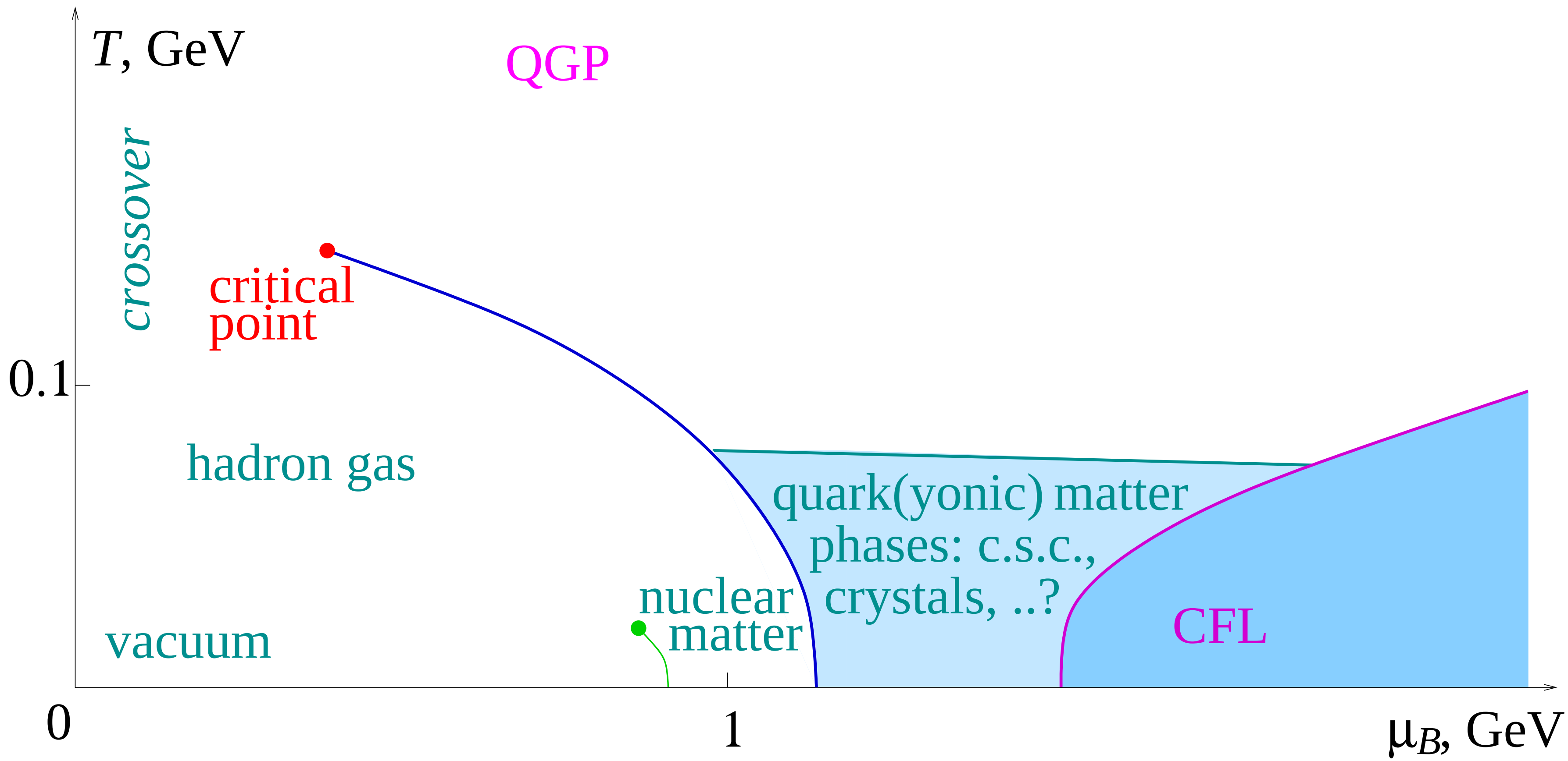
Deconfinement transition in QCD

- But there is a sense in which deconfinement does happen in QCD:

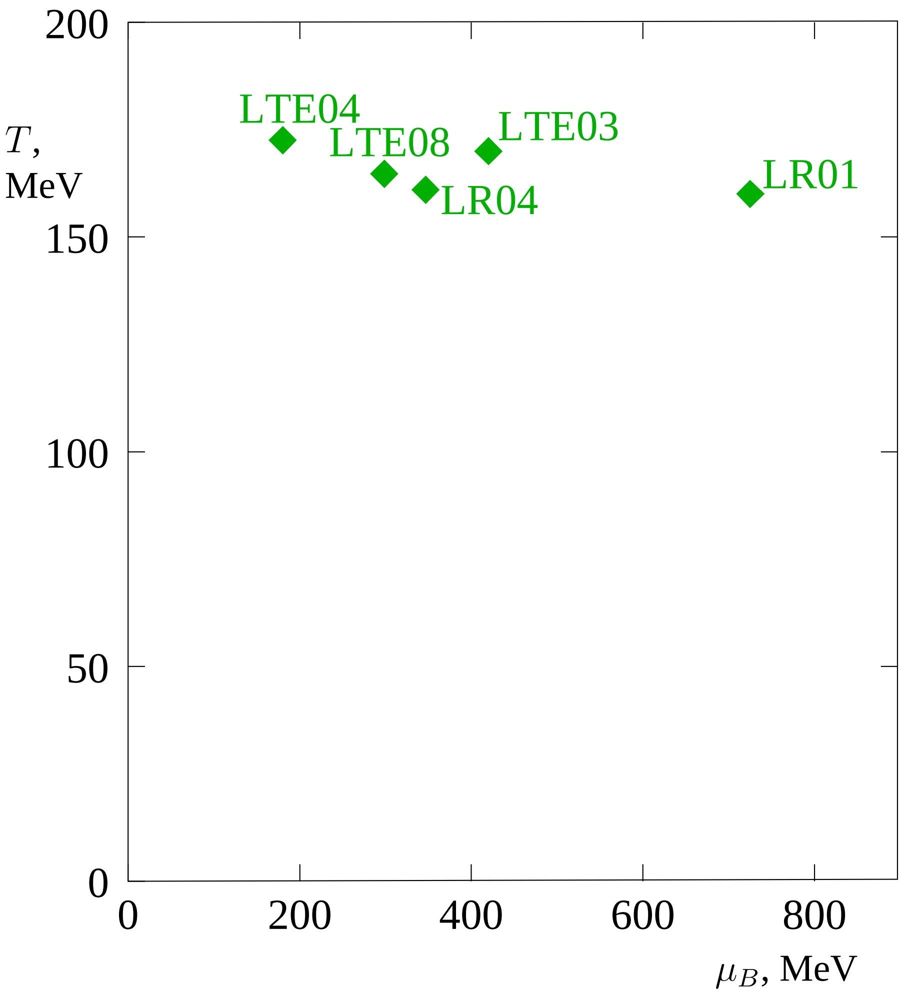


- s/T^3 – a measure of the number of degrees of freedom.
- gluons and quarks act as (count as) unconfined ("free") above T_c!
- NB: "free" as far as d.o.f. counting (s), but not necessarily as far as hydrodynamics (η).
- NB: even as $T \to \infty$ interaction energies are actually large ($\alpha_s T$), but the kinetic energies are larger still

Where exactly is the critical point?



Location of the critical point from the Lattice



- Systematic errors are not shown.

Sign Problem

- Thermodynamics is encoded in the partition function

$$Z = \sum_{\text{quantum states}} \exp\{-\beta(\mathcal{E} - \mu N)\} = \int \mathcal{D}(\text{paths}) \, \exp\{-S_E\}$$

  - S_E - action on a path in imaginary time τ from 0 to β.

- Usually, S_E - real. So $\int \mathcal{D}(\text{paths})\, e^{-S_E}$ - itself is a partition function for *classical* statistical system in $3+1$ dimensions. Monte Carlo methods work.

- Not so for $\mu \neq 0$.

$$e^{-S_E} = e^{-S_{\text{gluons}}} \det D_{\text{quarks}}.$$

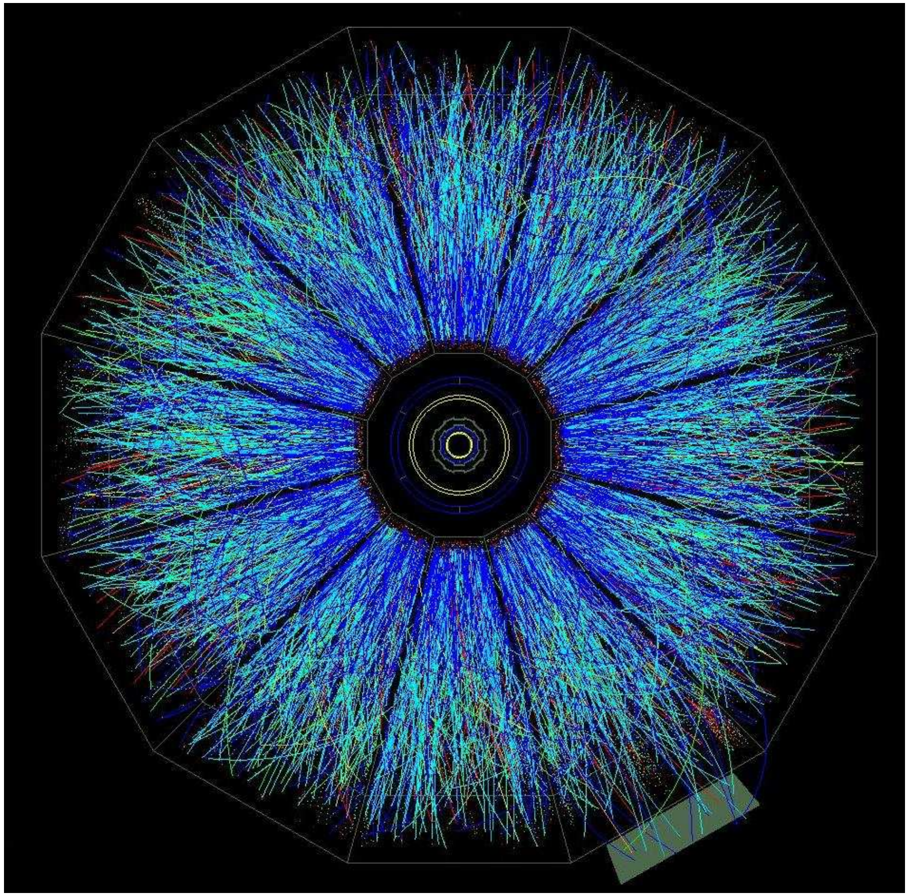
and $\det D_{\text{quarks}}$ - complex for $\mu \neq 0$.

Monte Carlo translates weight e^{-S_E} into probability and fails if S_E is not real.

- Recent progress based on various techniques of circumventing the problem:
  - Reweighting (use weight at $\mu = 0$);
  - Taylor expansion;
  - Imaginary μ;
  - ...

Heavy-ion collisions and the phase diagram

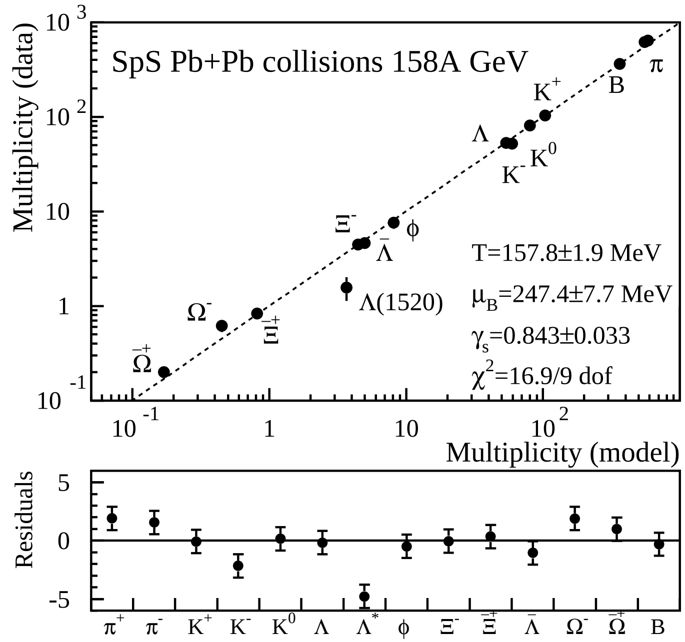
STAR@RHIC



an event

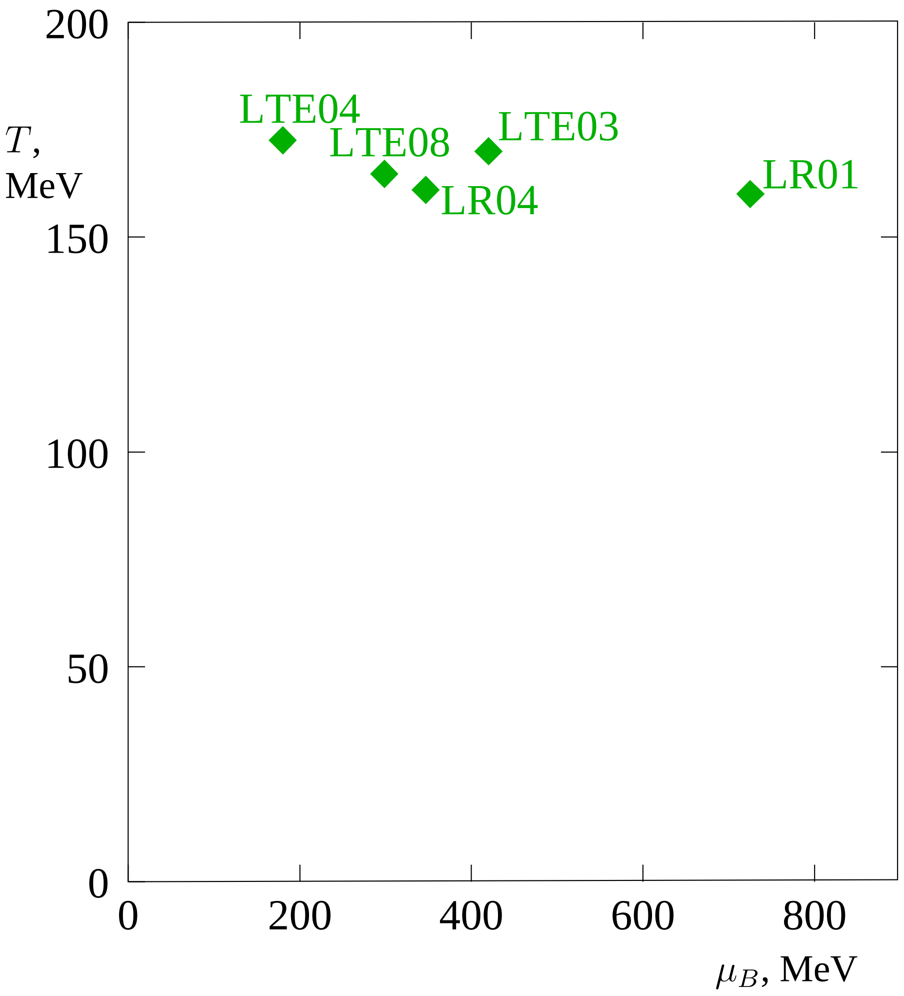
"Little Bang"

Final state is thermal



(from Becattini et al)

Location of the critical point vs freeze-out



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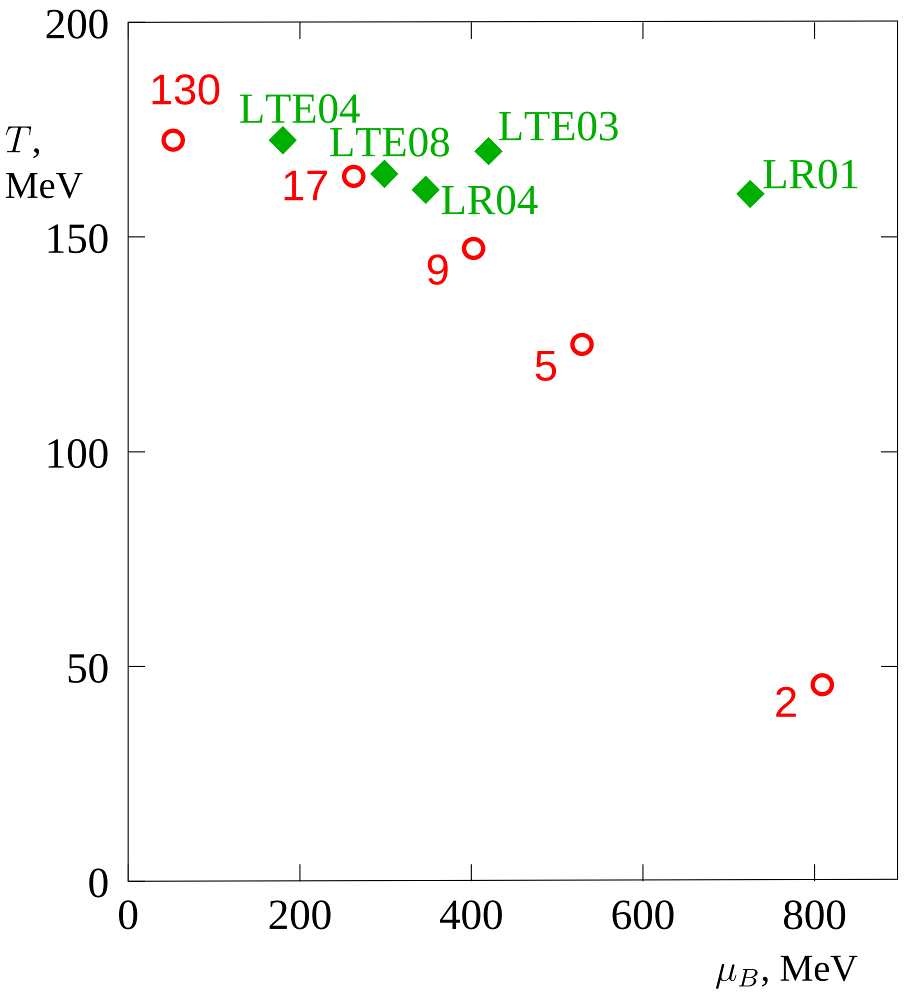

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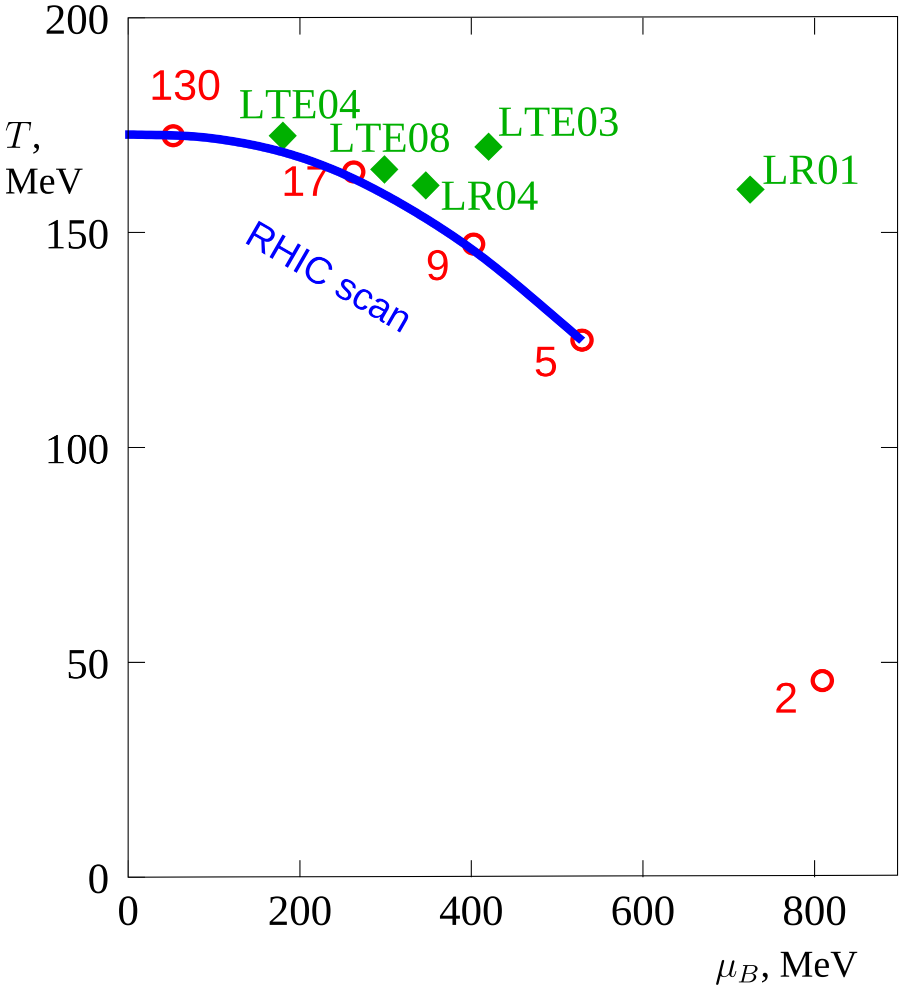
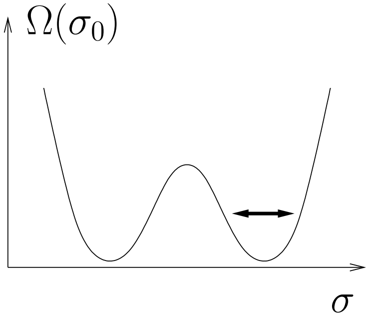


Needed:

- Experiments:
  - RHIC,
  - NA61(SHINE) @ SPS,
  - CBM @ FAIR/GSI
  - NICA @ JINR
- Improve lattice predictions, understand systematic errors.
- Understand critical phenomena in the dynamical environment of a h.i.c., develop better signatures

Critical mode and equilibrium fluctuations

- $\boxed{1}$

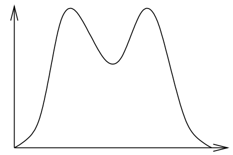
  

  $\mu > \mu_E$

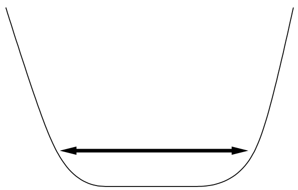
  Consider "soft mode" σ
  e.g., $\sigma \sim \bar\psi\psi - \langle\bar\psi\psi\rangle$
  Order parameter is $\int_V \sigma = \sigma_0$

  $\langle\sigma^2\rangle \sim (\Omega'')^{-1}$

  Einstein, 1910:
  $P(\sigma_0) \sim$ number
  of states with that σ_0
  i.e., e^S, or $e^{-\Omega/T}$

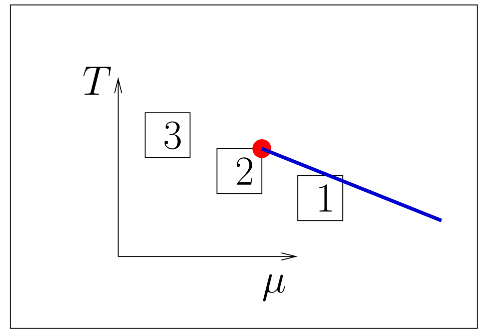
  $\boxed{2}$

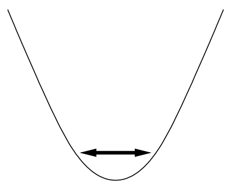
  $\mu = \mu_E$

  $(\Omega'')^{-1} \to \infty$

  large equilibrium fluctuations

  $\boxed{3}$

  $\mu < \mu_E$

- Magnitude of fluctuation and correlation length:

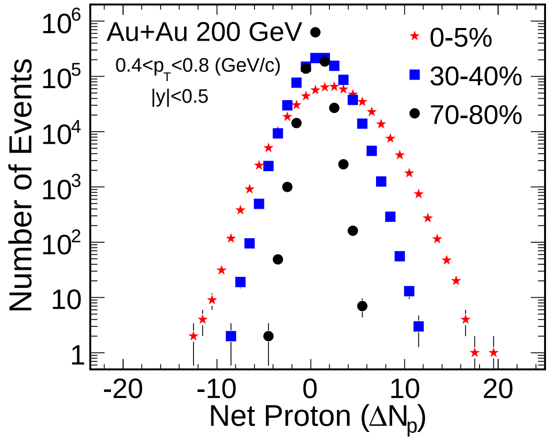
$$\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{0})\rangle \sim \begin{cases} e^{-|\boldsymbol{x}|/\xi} & \text{for} \quad |\boldsymbol{x}| \gg \xi \\ 1/|\boldsymbol{x}|^{1+\eta} & \text{for} \quad |\boldsymbol{x}| \ll \xi \end{cases}$$

$$\langle\sigma_{\boldsymbol{0}}^2\rangle = \int d^3\boldsymbol{x}\langle\sigma(\boldsymbol{x})\sigma(\boldsymbol{0})\rangle \sim \xi^{2-\eta}$$

critical singularity is a *collective* phenomenon

- σ or n_B or T^{00}? Because they mix, only *one* linear combination is critical.

Fluctuation signatures



- Experiments give for each event: multiplicities N_π, N_p, ..., set of momenta p, etc.
  These quantities fluctuate event-by-event.
  - Measure – sq. var., e.g., $\langle(\delta N)^2\rangle$,$\langle(\delta p_T)^2\rangle$.
- What is the magnitude of these fluctuations near the QCD C.P.? (Rajagopal-Shuryak-MS, 1998)
- *Universality* tells us how it grows at the critical point: $\langle(\delta N)^2\rangle \sim \xi^2$.
  Correlation length is a universal measure of the "distance" from the c.p.
  It diverges as $\xi \sim (\Delta\mu \text{ or } \Delta T)^{-2/5}$ as the c.p. is approached.
- Magnitude of ξ is limited $< \mathcal{O}($2–3 fm$)$ (Berdnikov-Rajagopal).

- "Shape" of the fluctuations can be measured: non-Gaussian moments.
  As $\xi \to \infty$ fluctuations become less Gaussian ($\xi \to \infty$ vs $N \to \infty$).
- Higher cumulants show even stronger dependence on ξ (PRL 102:032301,2009):

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}, \qquad \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \sim \xi^{7}$$

which makes them more sensitive signatures of the critical point.

Relation between σ fluctuations and observables

Consider example: fluctuations of multiplicity of pions (or protons).

- Free gas: $n^0_{\boldsymbol{p}}$ – fluctuating occupation number of momentum mode $\boldsymbol{p}$. Ensemble (event) average $\langle n^0_{\boldsymbol{p}} \rangle = f_{\boldsymbol{p}}$ and

$$n^0_{\boldsymbol{p}} = f_{\boldsymbol{p}} + \delta n^0_{\boldsymbol{p}}; \quad \langle \delta n^0_p \delta n^0_{\boldsymbol{k}} \rangle = f'_{\boldsymbol{p}} \delta_{\boldsymbol{pk}}; \qquad f_{\boldsymbol{p}} = (e^{\omega_{\boldsymbol{p}}/T} \mp 1)^{-1}; \; f'_{\boldsymbol{p}} \equiv f_{\boldsymbol{p}}(1 \pm f_{\boldsymbol{p}}).$$

- Couple these particles to σ field: $G\sigma\pi\pi$ (or $g\sigma\bar{N}N$). Think of $m^2 \equiv m_0^2 + 2G\sigma$ as "fluctuating mass". Then

$$\delta n_{\boldsymbol{p}} = \delta n^0_p + \frac{\partial f_{\boldsymbol{p}}}{\partial m^2} 2G\sigma = \delta n^0_p + \frac{f'_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}} \frac{G}{T} \sigma$$

- Using $\langle \delta n^0_p \sigma \rangle = 0$ and $\langle \sigma^2 \rangle = (T/V)\xi^2$.

$$\langle \delta n_p \delta n_{\boldsymbol{k}} \rangle = f'_{\boldsymbol{p}} \delta_{\boldsymbol{pk}} + \frac{1}{VT} \frac{f'_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}} \frac{f'_{\boldsymbol{k}}}{\omega_{\boldsymbol{k}}} G^2 \xi^2.$$

More formal derivation: PRD65:096008,2002

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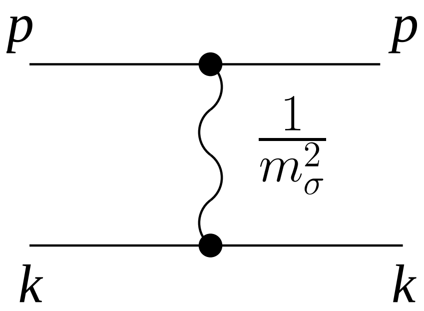
2-particle correlator as a 4-point function

- The 2-particle correlator measures 4-point function at $q=0$ (for $p \neq k$). Singularity appears at $q=0$ due to vanishing σ screening mass $m_\sigma \to 0$. (i.e., $\xi = 1/m_\sigma \to \infty$).



$$\langle \delta n_p \delta n_k \rangle_\sigma = \frac{1}{T} \frac{f_p(1+f_p)}{\omega_p} \frac{f_k(1+f_k)}{\omega_k} \frac{G^2}{m_\sigma^2}.$$

Check: $\langle \delta n_p \delta n_k \rangle = \langle n_p n_k \rangle - \langle n_p \rangle \langle n_k \rangle > 0$ — as in attraction. Attraction lowers the energy of a pair (making it more likely) by $\langle H_{\text{interaction}} \rangle \sim$ forward scattering amplitude.

- Consider baryon number susceptibility, which should diverge: $\chi_B \sim \xi^{2-\eta}$

$$\chi_B \sim \langle \delta B \delta B \rangle_\sigma = \langle (\delta N_p - \delta N_{\bar p} + \delta N_n - \delta N_{\bar n})^2 \rangle_\sigma = \langle \delta N_p \delta N_p \rangle_\sigma + \ldots$$

Each term on r.h.s. is $\sim \dfrac{1}{m_\sigma^2}$, $\qquad \Rightarrow \qquad \langle \delta B \delta B \rangle \sim 1/m_\sigma^2 = \xi^2$.

- - It is enough to measure protons $\langle \delta N_p \delta N_p \rangle$ (Hatta, MS, PRL91:102003,2003)

Higher moments (cumulants) of fluctuations

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

Ω – effective potential:

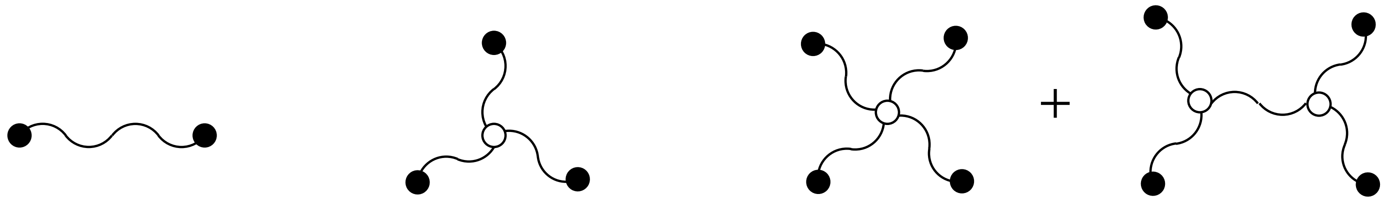
$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments of zero-momentum mode $\sigma_0 \equiv \int d^3x\, \sigma(x)/V$.

$$\kappa_2 = \langle\sigma_0^2\rangle = \frac{T}{V}\,\xi^2; \qquad \kappa_3 = \langle\sigma_0^3\rangle = \frac{2\lambda_3 T^2}{V^2}\,\xi^6;$$

$$\kappa_4 = \langle\sigma_0^4\rangle_c \equiv \langle\sigma_0^4\rangle - \langle\sigma_0^2\rangle^2 = \frac{6T^3}{V^3}\left[2(\lambda_3\xi)^2 - \lambda_4\right]\xi^8.$$

- Tree graphs. Each zero-momentum propagator gives m_σ^{-2}, i.e., ξ^2.



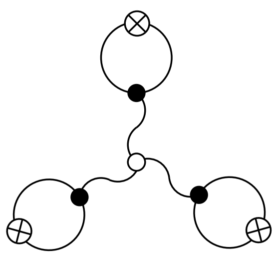
Moments of *observables*

- Example: multiplicity. Since multiplicity is just the sum of all occupation numbers, and thus

$$\delta N = \sum\nolimits_{\boldsymbol{p}} \delta n_{\boldsymbol{p}},$$

the cubic moment (skewness) of the pion multiplicity distribution is given by

$$\langle (\delta N)^3 \rangle = \sum\nolimits_{\boldsymbol{p}_1} \sum\nolimits_{\boldsymbol{p}_2} \sum\nolimits_{\boldsymbol{p}_3} \langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle \,, \qquad \text{where } \sum\nolimits_{\boldsymbol{p}} = V \int d^3\boldsymbol{p}/(2\pi)^3.$$

- 

$$\langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \left(\frac{G}{m_\sigma^2} \right)^3 \frac{v_{\boldsymbol{p}_1}^2}{\omega_{\boldsymbol{p}_1}} \frac{v_{\boldsymbol{p}_2}^2}{\omega_{\boldsymbol{p}_2}} \frac{v_{\boldsymbol{p}_3}^2}{\omega_{\boldsymbol{p}_3}}$$

$$v_{\boldsymbol{p}}^2 = \bar{n}_{\boldsymbol{p}}(1 \pm \bar{n}_{\boldsymbol{p}})$$

Similarly for $\langle (\delta N)^4 \rangle_c$.

- Since $\langle (\delta N)^3 \rangle$ scales as V^1 we suggest $\omega_3(N) \equiv \dfrac{\langle (\delta N)^3 \rangle}{\bar{N}}$ which is V^0.
- For more $\Rightarrow$ Christiana's talk.

Concluding remarks

- Phase diagram of QCD at nonzero T and μ_B is rich.
- Different corners are accessible by different methods.
- The interesting region: $T \sim \mu_B \sim 1\text{fm}^{-1}$ — is the most difficult:
  - Under active theoretical investigation: much progress in lattice approaches.
  - Still much to be done to narrow down the prediction for the critical point.
    Agreement between different approaches must be achieved.
    New methods are needed.
- Heavy ion collision experiments can discover the critical point by observing certain non-monotonous signatures — RHIC scan ($\sim$2010)
  or, for higher μ_B, – FAIR/GSI.